a few feet less than runs with the longer update interval. For trajectories and conditions tested, update rates approximately once per minute appear reasonable in order to attain arrival accuracies within a few seconds.

VI. Conclusions

The study indicates that a constant velocity controller system is an entirely feasible method for accomplishing 4-D aircraft control on a terminal or enroute environment. The command processor system is shown to provide an accurate solution to the velocity, heading, and bank angle commands. The system equations appear to be simple enough to implement on a small general-purpose airborne digital computer or to be solved as an adjunct to the ground controller's data processing and voice commands relayed to the pilot during radar vectoring instructions.

The simulation study indicates that for an aircraft with a typical autopilot and autothrottle, open-loop velocity control with refresh rates of approximately 1 min are adequate to accomplish sequencing accuracies in the order of a few seconds even when gusts and realistic errors in knowledge of the steady winds exist. Errors of this magnitude should significantly improve sequencing accuracy in terminal traffic control.

The results show that the utilization of programed bank angles based upon assumed steady winds significantly reduces dispersion on constant ground radius turns. The remaining dispersion is a result of procedure in execution and can be made acceptable by procedural standardization or route planning.

In this paper, the constant velocity terminal controller is demonstrated for a typical autopilot system. Its greatest asset, however, appears to be the wide class of systems for which it can be implemented. Further studies should be conducted to determine the accuracy capability for final control for other members of this class in order that an improved sequencing system be made available for air traffic control.

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JUNE 1974 J. AIRCRAFT VOL. 11, NO. 6

Stability and Control of Hingeless Rotor Helicopter Ground Resonance

Maurice I. Young* and David J. Bailey†

University of Delaware, Newark, Del.

The ground resonance instability of advanced helicopters employing hingeless rotors is examined on a broad parametric basis and a variety of conditions affording inherent stability are determined. Moderate levels of blade internal structural damping in conjunction with typical landing gear damping and stiffness characteristics are shown to be highly effective. This is shown to be a consequence of the offsets of the virtual flapping and lead-lag hinges together with the tuning of the elastically flapping and lead-lagging blades of a hingeless rotor system. Size and scale effects are also included by examining aerodynamically scaled designs which range in gross weight from 5,120 to 48,000 lb. Closed-loop stabilization of the ground resonance instability is considered by using a conventional helicopter swash-plate-blade cyclic pitch control system in conjunction with roll, roll rate, pitch, and pitch rate sensing. This also is shown to be highly effective due to the enormous control power inherent in the advanced hingeless rotor blade designs compared to that of the freely flapping, conventional rotors.

Received June 13, 1973; revision received April 8, 1974. Acknowledgment is made of the support of the U.S. Army Research Office, Durham, N.C., under Grant DA-ARO-D-31-1247G112. Based on a Master of Mechanical and Aerospace Engineering thesis by D. J. Bailey, prepared under the supervision of M. I. Young.

Index categories: Aircraft Handling, Stability, and Control; Aircraft Vibration; Aeroelasticity and Hydroelasticity.

*Professor of Mechanical and Aerospace Engineering. Associate Fellow AIAA.

†Graduate Student and Research Assistant, currently U.S. Army Transportation Engineering Agency, Fort Eustis, Va.

Nomenclature

C_e = landing gear equivalent viscous damping coefficient, lb/ft/sec
C_s = pneumatic shock strut viscous damping coefficient, lb/ft/sec
C_t = tire viscous damping coefficient, lb/ft/sec
CG = helicopter center of gravity
I_x = moment of inertia about x axis, slug-ft^2
K_e = landing gear equivalent spring rate, lb/ft
K_s = non-linear, pneumatic shock strut spring rate, lb/ft
K_t = tire spring rate, lb/ft
M = mass of helicopter

M_1 = control moment acting in lateral swashplate equation of motion, ft-lb
M_2 = control moment acting in longitudinal swashplate equation of motion, ft-lb
N = number of blades
T/W = thrust to weight ratio
XYZ = inertial coordinate system
db = decibels
e_1 = offset of virtual flapping hinge, ft
e_2 = offset of virtual lead-lag hinge, ft
h = distance between center of mass of helicopter and coordinate system axis, ft
t = lateral and roll coupling parameter
x,y,z = helicopter longitudinal, lateral and vertical displacements, ft
α_1, α_2 = helicopter pitch and roll angular displacements, rad
$\dot{\alpha}_1, \dot{\alpha}_2$ = helicopter pitch and roll rate, rad/sec
β_k = flapping angular displacement of kth blade, rad
γ_k = lead-lag angular displacement of kth blade, rad
δ = logarithmic decrement
ζ_1 = nondimensionalized (by rotor radius) displacement of virtual flapping hinge from rotor center of rotation, ft/ft
η_j = generalized fuselage and rotor system degrees of freedom
θ_i = constrained swashplate-blade pitch degrees of freedom
$\lambda_{\alpha 2}$ = percent of uncoupled critical roll damping
λ_{ll} = percent of uncoupled blade lead-lag damping
Ψ_k = azimuthal coordinate of the kth blade, rad
ω_β = out-of-plane or flapping frequency ratio, cycles/revolution
ω_{ll} = in-plane or lead-lag frequency ratio, cycles/revolution

Introduction

HELICOPTER ground resonance[1] is the destructive dynamic instability which couples the oscillatory motion of the aircraft on its landing gear with the divergent whirling of the rotor center of mass with respect to the center of the turning rotor.[2] This depends on and has greatly complicated the design and operation of these machines since the inception of blade articulation through use of mechanical hinges.[3] In recent years an intensive research and development effort within government and industry has focused on hingeless rotor helicopters with a view towards mechanical simplification, improved flying qualities, and greater aerodynamic cleanness. The approach being employed capitalizes on modern structural materials and technology which, in principle, permit the hingeless rotor blades to flap and lead-lag by flexing elastically, rather than by the use of mechanical hinges. In order to keep cyclic bending fatigue stress and blade weight within bounds, the in-plane or lead-lag hingeless blade fundamental natural frequency ratio, as a practical matter, inevitably falls within the range 0.6–0.9 cycles per revolution,[4] although frequency ratios as small as 0.5 or as great as 1.2 are possible.[5] As a consequence of this 0.6–0.9 range of frequencies, the ground resonance instability can still occur, since it depends on this frequency ratio being less than unity.

There now arises the added concern that the slight amounts of internal blade structural damping of hingeless rotor blades can cause the instability to be much more severe and difficult to control than in the articulated rotor case, where mechanical lead-lag dampers are a standard design feature. On the other hand, the elastic flapping of the hingeless rotor blades and the presence of large blade structural moments which are aeroelastically coupled to the fuselage oscillations on its landing gear, and the aforementioned relatively high-frequency ratio of hingeless blade lead-lag oscillations compared to those of conventional articulated rotors (0.2–0.4 cycles per revolution), present the possibility of significant, favorable alterations of the ground resonance stability characteristics. This is in contrast to centrally hinged, articulated rotors, where flapping motion has negligible effect on the instability.[6] Several recent investigations[7-9] have contributed to increased understanding of hingeless rotor helicopter ground resonance characteristics, but in each case were directed principally at design and development of a particular machine with its unique size, and structural and operational characteristics, rather than at broad development of parametric trends and general principles, particularly the possibilities for enhancing system stability by application of modern control engineering techniques.

In this study, the effects of the various design and operating parameters, which traditionally influence the ground resonance instability of articulated rotor helicopters such as landing gear stiffness and damping have been considered, but with the addition of the unique hingeless rotor helicopter parameters such as blade internal damping and virtual hinge locations. The effect of scale on stability is investigated by considering aerodynamically similar designs which range in gross weight from 5,120 lb to 48,000 lb. In view of the enormous control power available with a hingeless rotor due to its structural characteristics and the possible need for or desirability of full artificial stabilization or stability augmentation of certain design configurations or operating conditions, a closed-loop stabilization approach is also investigated. It is viewed as an evolutionary approach which would employ a conventional helicopter swashplate type of control system of blade collective and cyclic pitch. A variety of output variables and their derivatives are examined as possible sources of closed-loop feedback information for control actuation. The roll and the roll rate variables are seen to be highly effective. The dynamics of cyclic and collective pitch change are also examined[10] as part of such a closed loop stabilization system for ground resonance where the control process is seen to be that of a multiple input-multiple output, interacting control system.[11]

A detailed parametric study of the ground resonance stability boundary is carried out using a standard eigenvalue routine. The parameter combinations which can result in the ground resonance instability are examined with a view towards comparing designs with inherent stability with those that are a result of artifical stabilization. Finally, those combinations of design, operating, and stability augmentation parameters which point towards hingeless rotor type aircraft which are free of the ground resonance instability are obtained.

Analysis

The analysis is carried out with the objective of developing a broad understanding of the influence of the principal design and operating parameters on the system stability. Consequently, the degrees of freedom chosen for the analytical model are those which can be expected to be common to all hingeless rotor helicopter designs, irrespective of size and gross weight, operational requirements, or specific structural design approaches.

The fuselage body degrees of freedom are taken as those which would be representative of the ground oscillations of a single rotor helicopter on a three point, conventional oleo-shock strut type of landing gear. These then follow as the lateral, longitudinal, and vertical translational degrees of freedom and the angular roll and pitch degrees of freedom. A yawing degree of freedom is not included, since it is deemed an unnecessary and unproductive complication. This follows from the large yawing inertia of the body, the close proximity of the aircraft center of gravity to the two main landing gear and the rotor thrust line, the net effect of which is to virtually decouple the yawing freedom from the others, and thereby effectively eliminates its influence on the ground resonance instability. The body degrees of freedom are illustrated in Fig. 1.

The landing gear type and arrangement used in the analysis are viewed as typical, but by no means universal. However, the effective spring and viscous damping restraints which are arrived at in the landing gear analysis are sufficiently broad in character to be representative of the many different landing gear systems currently in use. The two most prevalent systems are the skid type, and pneumatic shock strut and tire type configurations. Since

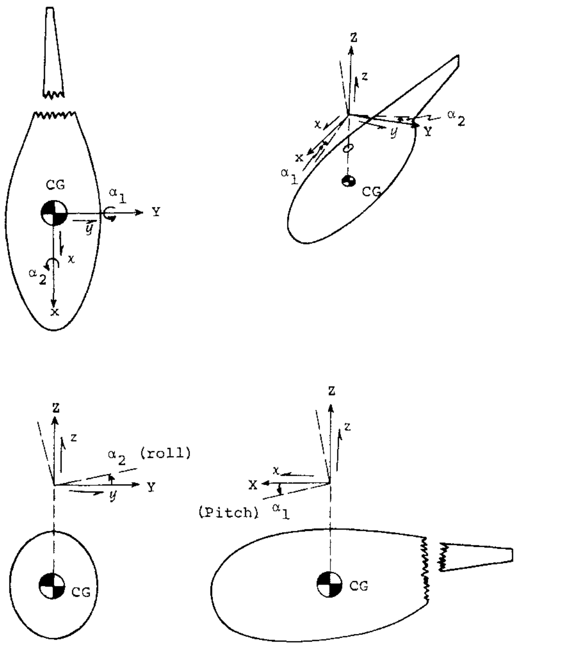

Fig. 1 Fuselage degrees of freedom. X, Y, Z, α_1, α_2 = displacements. XYZ = coordinate system.

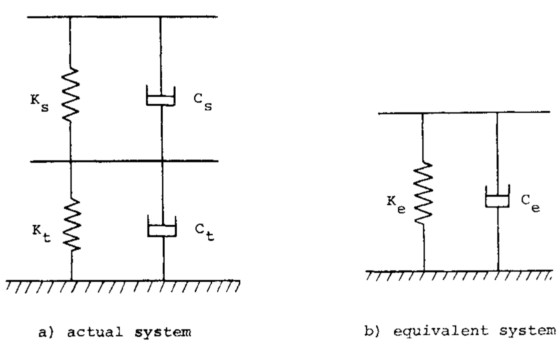

Fig. 2 Landing gear model.

the skid-type landing gear represents a special case of the more general shock strut and tire formulation, an analytic model of the latter has been employed. This formulation has the added advantage of permitting various effects, such as the shock strut damping, the nonlinear pneumatic spring rate, and the combined spring rate of the tire and landing surface, to be more easily studied.

The helicopter suspension system consists of a conventional three point landing gear configuration. The two primary supports, located slightly to the rear of the helicopter center of gravity and on each side of the helicopter, supply the major reactions on the aircraft. A third support, located at the fuselage nose, is employed principally for static stability purposes and as such does not exhibit the same properties as the primary supports. Each support consists of a pneumatic shock strut connected in series with a tire in the usual manner. Thus each support is modeled as shown in Fig. 2a as a series-parallel combination of two springs and two dashpots in each of the three principal directions. Because of the inherent complexity in determining the reactions from such a system in conjunction with the already complex motion of the aircraft, a greatly simplified but equivalent system consisting of a spring and dashpot in parallel shown in Fig. 2b is employed. These two systems are related by matching their frequency response over the interval of interest. This is conveniently done by the use of a Bode analysis.[12,13] The results which are shown in Fig. 3 indicate excellent agreement between the two models.

In modeling the pneumatic characteristics of the shock

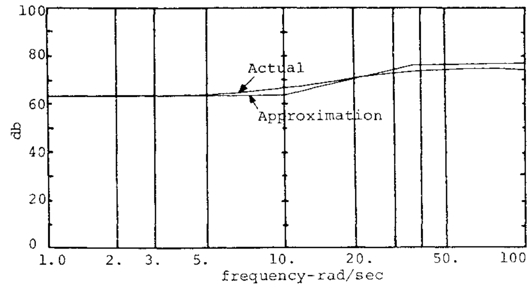

Fig. 3 Bode diagram of landing gear frequency response.

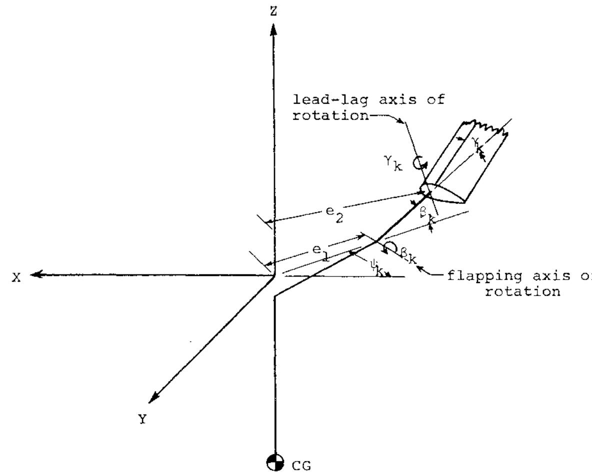

Fig. 4 Virtual hinge, blade degrees of freedom.

strut, it was found that the nonlinear spring rate is principally a function of the effective load acting on each support.[10,13] As such, the nonlinear characteristics of the landing gear, exemplified in terms of the thrust to weight ratio or effective load on the landing gear, is employed as a principal parameter in determining the characteristics of the system.

The hingeless rotor blades are flexible, cantilever structures which flap elastically in oscillations normal to the plane of rotation and lead-lag elastically in the plane of rotation. A generalized coordinate, normal mode type of analysis[14] could be employed effectively for the structural dynamic aspects. However, this does not lend itself well to determination of the aerodynamic forces and moments which play a central role in the stabilization process because of the blade bending curvature during the oscillations. Consequently, the concept of virtual springs and hinges[15,16] for the flapping and lead-lag oscillations of the blade is used, where quasi-rigid body blade motions are introduced to replace the continuous, elastic bending deformations of the real blades. These degrees of freedom are illustrated in Fig. 4.

The blade pitch changes are treated as constrained degrees of freedom in the stability analysis. That is the blade pitch can be changed collectively or cyclically by displacement or tilting of a swashplate mechanism. In the open-loop case this is done by the pilot displacing the collective or cyclic pitch control sticks. This results in a transient response of the aircraft on its landing gear by altering the aerodynamic forces and moments produced by the hingeless rotor. Since it takes the form of a reference input or external disturbance, it has no effect on the system stability as long as these disturbances are reasonably small. In the closed-loop case the aircraft roll position, roll rate, pitch position, and pitch rate are sensed and used to drive a system of swash-plate actuators with a view towards employing the enormous control power inherent in the cantilever blade design of hingeless rotor systems. This yields full stabilization, if required, or aug-

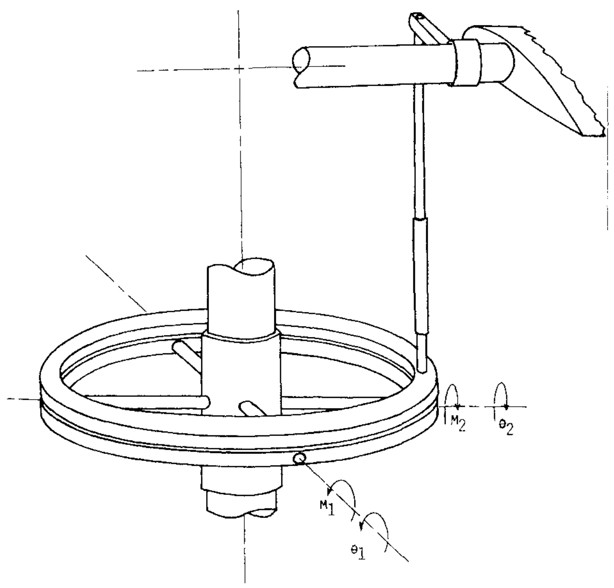

Fig. 5 Schematic of swashplate-blade pitch control.

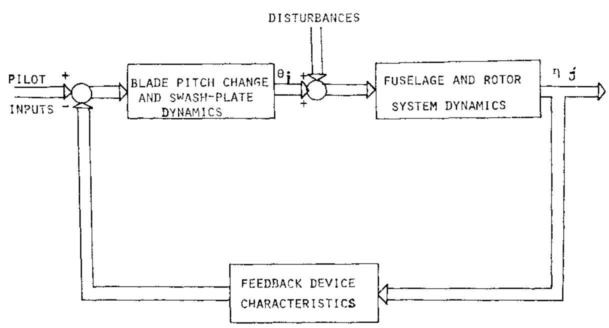

Fig. 6 Closed-loop block diagram.

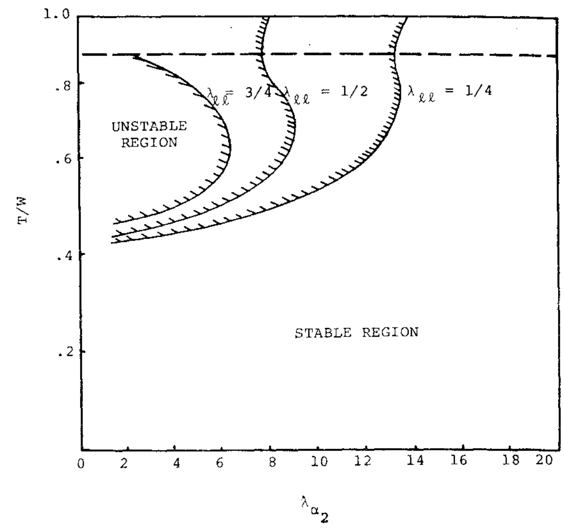

Fig. 7 Effect of roll damping on stability boundary.

ments the inherent stability of the system when design and operating conditions permit. The swashplate-blade pitch change arrangement and the system block diagram are shown schematically in Figs. 5 and 6. More sophisticated closed-loop control system arrangements offer the possibility of enhanced performance and optimization of the system at the expense of complexity or possible reduction in reliability. For example, an inner control loop on rotor blade bending deflections by strain gage techniques, as well as sensing of body translational displacements and velocities offer interesting possibilities which are considered in Ref. 10.

The combination of the fuselage landing gear and the rotor blade systems yields $5+3N$ freedoms in the closed-loop case and $5+2N$ freedoms in the open-loop case where the blade number N is at least four. The minimum number of four blades follows from the possibilities of a dynamic instability unique to two-blade systems[17] and resonant amplification of three blade aerodynamic loadings in the case of three blades[18] which must be avoided by using a minimum of four blades in a hingeless rotor system.

The number of blade freedoms is reduced by introduction of quasi-normal coordinates to describe the rotor motions.[19,20] This approach reduces the complexity of the analysis by eliminating all blade motions which do not couple with the body in a coherent manner during open- and closed-loop oscillations. These coordinates describe the various significant patterns of blade motion by five degrees of freedom in the open-loop case. These are the rotor cone vertex angle, the lateral and longitudinal tilt of the rotor cone, and the lateral and longitudinal displacements of the blade system center of gravity with respect to the geometric center of the rotor (due to lead-lag motion in the rotating frame of reference). In the closed-loop case three freedoms are added through the displacements of the swashplate for blade collective pitch changes and by the angular tilting of the swashplate for blade lateral and longitudinal cyclic pitch changes.

The analysis proceeds assuming that the rotor system has four blades. This leads to a final quasi-normal coordinate model which has ten degrees of freedom for the open-loop case and thirteen for the closed-loop case. These equations of motion are then reduced to a canonical form suitable for application of a standard digital computer routine for determining the complex eigenvalues and eigenvectors of the system. In effect, twice the number of first-order, linear differential equations with constant coefficients result. This is a twenty-sixth-order system in the closed-loop case, if ideal actuators are assumed. As more realistic models of the control hardware are employed (due to leakage across hydraulic seals, imperfect relays, amplifier frequency response characteristics, etc.) the order of the system increases further.

Discussion of Numerical Results

In order to develop insight into the nature of the ground resonance instability as it might occur for a typical helicopter employing a hingeless rotor, a reference case based on the S-58 helicopter[21] is considered first. The rotor is modeled as one with four hingeless blades with a flapping frequency ratio of 1.15 cycles per revolution, and a lead-lag frequency ratio of 0.70 cycles per revolution at a rotor tip speed of 650 fps. The wheels are first assumed to be locked, preventing the aircraft from rolling freely in a longitudinal direction. The uncoupled lateral and longitudinal translation modes of the aircraft are assumed to have five percent and three percent of critical damping, respectively, as a result of tire hysteresis losses. As the thrust-to-weight ratio is varied from zero to unity the vertical loading on the landing gear decreases. The stability of the small, coupled oscillations about a series of initial steady states determined by the thrust to weight ratio (T/W) is then studied as a function of oleo-shock strut damping for several small, but typical values of blade hysteresis lead-lag damping. Both damping parameters are expressed in terms of percent of equivalent viscous critical damping.

The unstable mode of oscillation is found in all cases to be dominantly a fuselage rolling mode with a small amount of lateral translation coupling, and still lesser amounts of pitching and longitudinal motion. Release of the brakes, permitting the aircraft to move freely longitudinally, has a slightly stabilizing effect, but of minor importance compared to the influence of oleo-shock strut

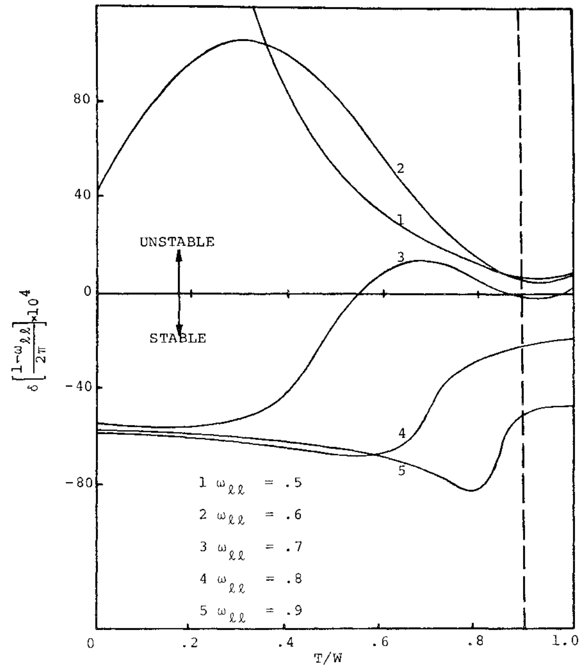

Fig. 8 Effect of varying lead-lag frequency ratio.

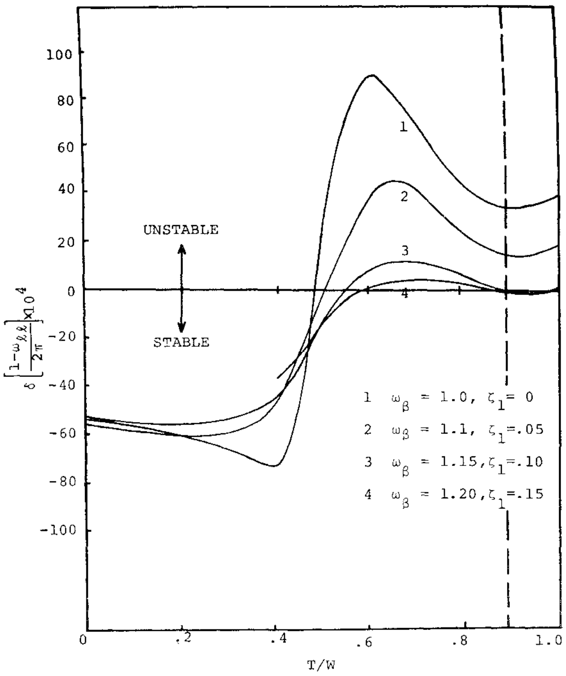

Fig. 9 Effect of flapping frequency ratio and offset of flapping virtual hinge.

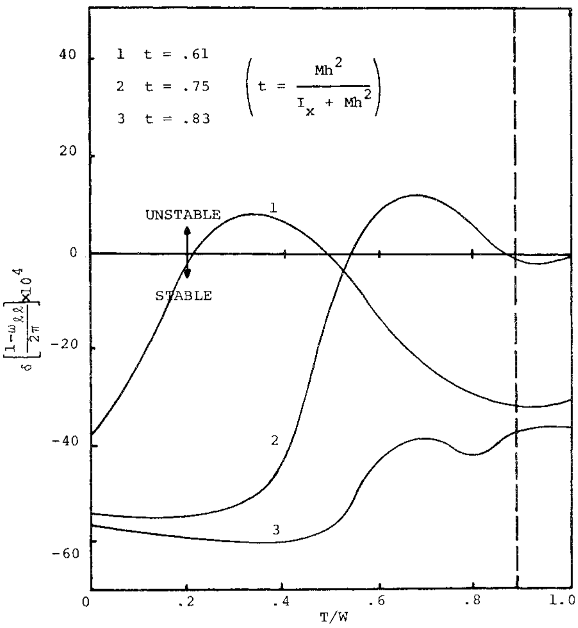

Fig. 10 Effect of coupling lateral and roll motion.

damping and blade internal damping. The numerical results of this portion of the study with brakes on are presented in Fig. 7. Equivalent viscous damping of the uncoupled rolling mode expressed in percent of critical damping is taken as the abscissa, while thrust-to-weight ratio is the ordinate. The horizontal dash line at T/W = 0.9 is a visual reminder that this is an unrealistic condition and that the stability data beyond this value is probably unreliable, since the analytical modeling of the landing gear depends on the questionable assumption of an initial steady-state for thrust-to-weight ratios greater than nine tenths. The aircraft is, of course, in the transient condition of landing or takeoff.

It is seen that if blade hysteresis damping should be equivalent to one percent of critical lead-lag damping, then slight amounts of oleo-damping of the rolling mode produce stable oscillations. If the blade internal damping is as little as one quarter a percent of critical, stability can still be achieved for all thrust to weight ratios, if roll damping is equivalent to fourteen percent of critical damping. Internal blade damping of one percent or greater is found to eliminate the instability entirely, if only slight amounts of landing gear damping are available, for example from tire hysteresis. Thus the ground resonance instability for the reference case is found to be quite mild and easily eliminated with the moderate amounts of blade and landing gear damping normally present.

In order to understand the influence of the tuning of a hingeless rotor on this desirable result, the lead-lag frequency ratio is varied about the reference frequency ratio of 0.7 cycles per revolution as the flapping frequency ratio is held constant at 1.15 cycle per revolution. Blade damping is taken at one half percent of critical while roll damping is held fixed at eight percent of critical. Figure 8 shows the effect of this tuning on the unstable mode by plotting the log decrement of this mode versus thrust to weight ratio. It is seen that increasing the lead-lag frequency ratio above 0.7 makes the system stable, while decreasing it below this reference value makes it progressively more unstable. Figure 9 considers the effect of the offset of the virtual flapping hinge and tuning of the flapping frequency ratio on the instability with respect to the reference case. It is seen that a flapping frequency ratio of 1.0 corresponding to a conventional, articulated rotor is considerably more unstable than the reference case. It is seen that increasing the offset and frequency ratio to progressively higher values is beneficial and stabilizing although tending to reach a point of diminishing returns at a flapping frequency ratio of 1.20 cycles per revolution.

Size and scale effects are first investigated by considering the coupling of the lateral and rolling motion as the distance between the rotor hub and the center of gravity of the aircraft is varied with respect to the reference case, where it was assumed to be at a distance of seven feet. As this distance is decreased to five feet, the instability is observed to change in relationship to the thrust-to-weight ratio, but not in general character. On the other hand, as the coupling increases by increasing the distance to nine feet, there is a stabilizing effect. This is illustrated in Fig. 10. This result can be understood in terms of the coupled rolling natural frequency, which tends to decrease as this distance increases. Thus if the lead-lag natural frequency ratio is held fixed at 0.7, stability can be improved by de-

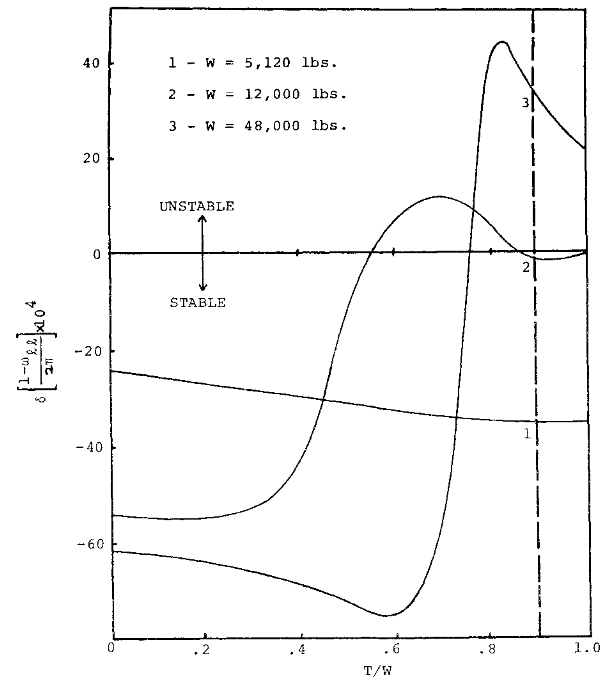

Fig. 11 Effect of size and gross weight.

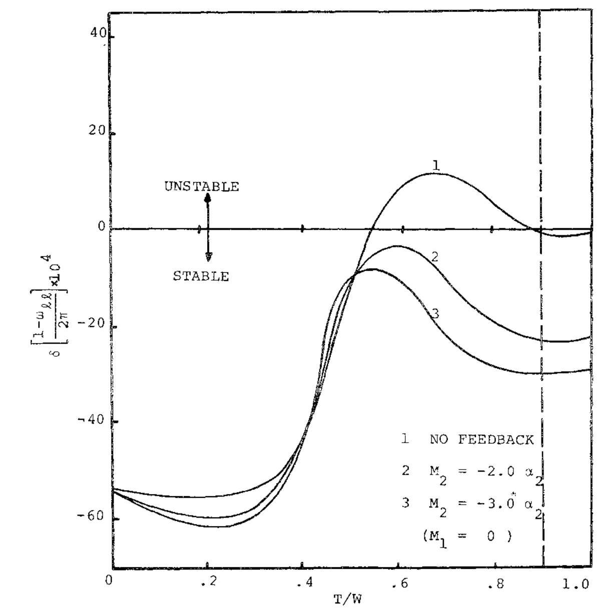

Fig. 12 Effect of proportional roll feedback.

tuning the fuselage coupled rolling mode to a lower frequency. This result is typical of all helicopter ground resonance instabilities.

The influence of large size and scale changes is considered by studying the stability of two additional hingeless rotor helicopters of 5,120 and 48,000 lb, respectively, which are obtained from the reference case by aerodynamic scaling. That is the rotor diameter and over-all proportions of the aircraft were altered to accommodate the gross weight changes at the same mean rotor lift coefficient. It is seen in Fig. 11 that aircraft smaller than the reference case of 12,000 lb tend toward inherent stability with the blade tuning and nominal amounts of damping assumed. On the other hand, the relatively heavy machines tend to a more severe instability at slightly higher thrust-to-weight ratios than the reference case, but still well within the range of achieving inherent stability with moderate amounts of blade hysteresis damping and oleo-shock strut damping of the unstable, coupled rolling mode.

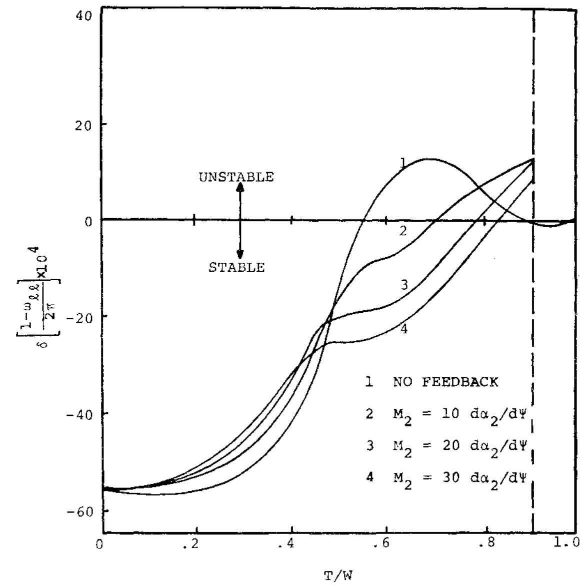

Fig. 13 Effect of derivative roll feedback.

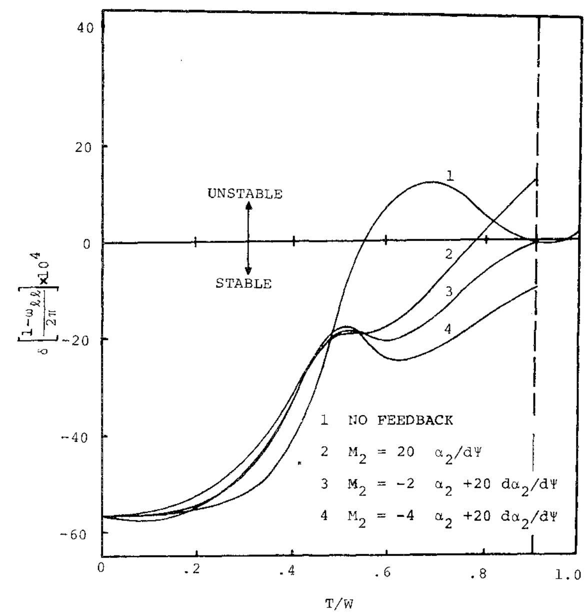

Fig. 14 Effect of combined proportional and derivative roll feedback.

As an alternative or as a supplement to parameter selection which results in stable oscillations, closed-loop feedback control is considered. Since proportional control action (at least qualitatively) alters the frequency of oscillation of simple systems by adding or subtracting a virtual spring effect, depending on whether feedback is negative or positive, the reference case was used as a basis for investigating this possibility. Figure 12 shows the effect of proportional roll feedback and control action (in this case, positive feedback is actually employed) in detuning an unstable coupling by depressing the critical fuselage roll mode frequency. It is seen that this is very effective in stabilizing the system. It should be noted that in the case of other design reference parameters, proportional feedback and control action of opposite sign might be beneficial, if the detuning of the critical fuselage roll frequency required increasing, rather than decreasing. The application of this control action is deemed beneficial, but is best decided on an ad hoc basis.

A more conventional use of feedback control is considered in Fig. 13 which shows the effect of negative feedback with derivative or rate control action. This tends to augment the damping of the critical fuselage rolling mode. This is seen to be highly effective also and, at least to a first approximation, is interchangeable with oleo-shock strut damping of the unstable roll mode.

A logical extension of the foregoing application of feedback control to the stability of ground resonance is the blending of both proportional and derivative control action. In this case, the critical roll mode can be both detuned and damped to approach an optimum. This is shown to be the case in Fig. 14. Here the system is made progressively more stable over the entire range of thrust-to-weight ratios. It is not the intention here to optimize the stability boundary, but to show that this is possible even with small values of blade internal hysteresis damping and the normal amounts of landing gear damping of the reference case. In view of the relatively unimportant influence of the pitching, and longitudinal degrees of freedom for the reference case, pitch and pitch rate feedback, and control action was not deemed effective. However, this remains a potentially useful and important tool in the event that special design or operational requirements modify the open-loop system.

Conclusions

It is concluded that helicopters employing hingeless rotors are less prone to the ground resonance instability than comparable aircraft with conventional articulated rotors. This stems primarily from the relatively high-frequency ratio of blade lead-lag oscillations of the order of 0.7–0.8 cycles per revolution and flapping frequency ratios of the order of 1.1–1.2 cycles per revolution. In this event achievable levels of blade internal hysteresis damping of one-half to one percent of critical damping are likely to be sufficient for inherent stability, providing the landing gear damping of the critical rolling mode is of the order of five to ten percent of critical. Size and scale effects are seen to be of some importance, but present no difficulty. That is relatively small helicopters of the order of 5000 lb gross weight are likely to be inherently stable with no special effort in this regard. Helicopter designs with an order of magnitude increase in gross weight to 50,000 lb are less stable than the small ones, but appear to present no difficulty. Proper tuning of hingeless blade frequencies together with moderate amounts of internal hysteresis damping tend to yield inherent stability.

The enormous control power inherent in hingeless rotor blade designs makes feedback control an effective means of augmenting or substituting for inherent stability of the system. For example, proportional control action can be employed to avoid instability by detuning the critical roll mode frequency while derivative control action can be employed to damp this mode. A suitable blending of these can be used to achieve an optimum design with respect to the ground resonance instability.

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